I Semester B.A./B.Sc. Examination, Nov./Dec. 2018
(Semester Scheme) (2011-12 and Onwards) (N.S.)
(Repeaters – Prior to 2014-15)
MATHEMATICS – I

Time : 3 Hours Max. Marks : 100

***Instruction** : Answer **all** questions.*

I. Answer **any fifteen** questions. **(15×2=30)**

1) Reduce the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$ to Echelon form.

2) State Cayley-Hamilton theorem.

3) Verify the equations $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$ for consistency.

4) Find the eigenvalue for the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

5) Find the n^{th} derivative of cos 2x.

6) Find the n^{th} derivative of e^{2x}.

7) If $u = x \tan y + y \tan x$, find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

8) State Euler's theorem for homogeneous functions.

9) Find $\frac{du}{dt}$ if $u = e^x \sin y$ where $x = \log t$, $y = t^2$.

10) If $x = r\cos\theta$, $y = r\sin\theta$, find $\frac{\partial(x.y)}{\partial(r.\theta)}$.

11) Evaluate $\int_0^{\pi/2} \cos^6 x \, dx$.

12) Using reduction formula, evaluate $\int_0^{\pi/4} \tan^6 x\, dx$.

13) Find the direction cosines of a line which makes angles 90°, 60° and 30° with the co-ordinate axes.

14) Show that the lines whose direction ratios are (2, 3, 4) and (1, –2, 1) are at right angles.

15) Find the projection of the line segment AB on CD where A = (3, 4, 5), B = (4, 6, 3), C = (–1, 2, 4) and D = (1, 0, 5).

16) Show that the planes $2x - 4y + 3z + 5 = 0$ and $10x + 11y + 8z - 17 = 0$ are perpendicular.

17) Find the angle between the planes $x - y + z = 6$ and $2x + 3y - 3z + 5 = 0$.

18) Find the equation of the sphere whose centre at (2, –3, 4) and radius equal to 5 units.

19) If a right circular cone has three mutually perpendicular generations show that the semi vertical angle is $\tan^{-1}\sqrt{2}$.

20) Find k if the spheres $x^2 + y^2 + z^2 + 6y + 2z + k = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ cut orthogonally.

II. Answer **any two** questions. **(2×5=10)**

1) Reduce the given matrix to normal form and hence determine its rank

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}.$$

2) Using Cayley-Hamilton, find A^3 if $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

3) Verify the system of equations for consistency and if consistent solve them $x + y - 2z = 5$, $x - 2y + z = -2$ and $-2x + y + z = 4$.

4) Find the eigenvalues and the eigenvectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

III. Answer **any four** questions. (4×5=20)

1) Find the n^{th} derivative of $\frac{x^4}{(x-1)(x-2)}$.

2) If $y = \sin(m \sin^{-1}x)$ prove that $(1 - x^2)y_{n+2} = (2n + 1)xy_{n+1} + (n^2 - m^2)y_n$.

3) If $z = \sin(ax + y) + \cos(ax - y)$ prove that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$.

4) State and prove Euler's theorem.

5) If $u = \log \frac{x^4 + y^4}{x - y}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$.

6) If $u = x + y + z$, $v = y + z$, $w = z$, find $\frac{\partial(u.v.w)}{\partial(x.y.z)}$.

IV. Answer **any two** questions. (2×5=10)

1) Evaluate $\int_0^{\pi} x \sin^7 x \, dx$.

2) Obtain the reduction formula for $\int \cos^n x \, dx$.

3) Using Leibnitz's rule of differentiation under integral sign, evaluate $\int_0^1 \frac{x^{\alpha} - 1}{\log x} dx$ where α is a parameter.

V. Answer **any four** questions. (4×5=20)

1) Find the angles of the triangle ABC where A = (1, –2, –3), B = (2, –3, –1) C = (3, –1, –2).

2) Find the value of 'a' such that the points (3, 2, 1) (4, a, 5) (4, 2, –2) and (6, 5, –1) are coplanar.

3) Find the co-ordinates of the foot of the perpendicular drawn from the point (1, 2, 3) to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$.

4) Find the equation of the plane which contains the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{5}$ and parallel to the line $\frac{x}{4} = \frac{y}{1} = \frac{z}{2}$.

5) Find the angle between the plane $x - 3y + 2z - 7 = 0$ and the line $x - 2y + 3z + 1 = 0 = 3x + y + 2z + 2$.

6) Find the shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{-1}$.

VI. Answer **any two** questions. **(2×5=10)**

1) Find the equation of the sphere intersecting the spheres $x^2 + y^2 + z^2 + x - 3z - 2 = 0$ and $2x^2 + 2y^2 + 2z^2 + x + 3y + 4 = 0$ orthogonally and passing through the points (0, 3, 0) and (–2, –1, –4).

2) Find the equation to the right circular cone whose vertex is (1, –1, 2) axis along the line $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{-2}$ and the semi vertical angle 45°.

3) Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1}$.
